The dihadron fragmentation function and its evolution

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Inclusive hadron production cross sections in e^+e^- collisions have turned out to be one of the many successful predictions of perturbative QCD. For reactions at an energy scale much above Λ_{QCD} one can factorize the cross section into a short-distance parton cross section which is computable order by order as a series in $\alpha_s(Q^2)$; and a long-distance phenomenological object (the single hadron inclusive fragmentation function) which contains the non-perturbative information of parton hadronization. These fragmentation functions can be defined in an operator formalism and hence are valid beyond the perturbative theory. They, however, cannot be calculated perturbatively and have to be, instead, inferred from experiments. The definition of these functions affords them the mantle of being universal or process-independent. Once these functions are measured at a given energy scale, they can be predicted for all other energy scales via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations

In this article [1], we will be concerned with the double inclusive fragmentation function $D_q^{h_1,h_2}(z_1,z_2,Q^2)$ or the dihadron fragmentation function in e^+e^-annihilation. The operator definition of this function is not merely a trivial extension of the single hadron case; there are no straightforward sum rules connecting it to single inclusive fragmentation functions.

In the limit of very large Q^2 of the reaction, we invoke the collinear approximation. Under this approximation, at leading twist, we demonstrated the factorization of the two-hadron inclusive cross section into a hard total partonic cross section σ_0 and the double inclusive fragmentation function $D_q^{h_1h_2}(z_1,z_2)$ (see Ref. [1] for details),

$$\frac{d^2\sigma}{dz_1dz_2} = \sum_q \sigma_0^{q\bar{q}} \left[D_q^{h_1h_2}(z_1,z_2) + D_{\bar{q}}^{h_1h_2}(z_1,z_2) \right]. \quad (1)$$

In the above equation, the leading order double inclusive fragmentation function of a quark is obtained as

$$D_q^{h_1,h_2}(z_1,z_2) = \int \frac{dq_\perp^2}{8(2\pi)^2} \frac{z^4}{4z_1z_2} \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4x e^{i\mathbf{p}\cdot\mathbf{x}}}{2\mathbf{n}\cdot\mathbf{p}_h} \sum_S$$
$$\mathbf{Tr}\left[\not{n} \langle 0|\psi_q^\alpha(x)|p_1p_2S\rangle\langle p_1p_2S|\bar{\psi}_q^\beta(0)|0\rangle \right] \delta\left(z - \frac{p_h^+}{p^+}\right). \quad (2)$$

In the above equation $z = z_1 + z_2$, $\mathbf{p}$ represents the momentum of the fragmenting parton, $\mathbf{p}_h$ is the sum of the momenta of the two detected hadrons *i.e.* $\mathbf{p}_1 + \mathbf{p}_2$ and $\mathbf{n}$ is a light-like null vector. The sum over S indicates a sum over all possible final hadronic states. The above equation may be represented by the diagrams of the cut vertex notation as that in Fig. 1. Note that all transverse momentum $q_\perp$ up to a scale $\mu_\perp$ have been integrated over into the definition of the fragmentation function. Hadrons with transverse momenta $\geq \mu_\perp$ may not emanate from the fragmentation of a single parton.

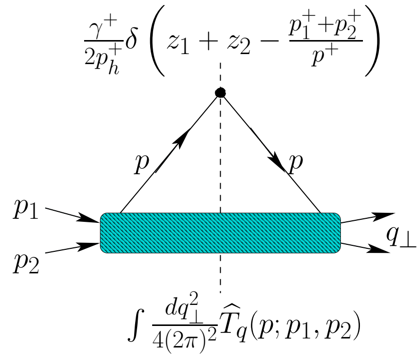

FIG. 1: The cut-vertex representation of the dihadron fragmentation function.

We also derive the DGLAP evolution of the dihadron fragmentation functions by computing the double inclusive cross section at next to leading order (NLO). In the remaining, we will focus on the non-singlet (NS) fragmentation functions for simplicity. In this case, the contribution from dihadron gluon fragmentation drops out. Summing over the contributions from soft gluon bremsstrahlung and independent single fragmentation following a semihard split we obtain the NLO contribution to the NS dihadron fragmentation function as,

$$\begin{aligned} D_{NS}^{h_1,h_2}(z_1,z_2,Q^2) &= D_{NS}^{h_1,h_2}(z_1,z_2) \\ &+ \frac{\alpha_s}{2\pi}\int^{Q^2}\frac{dp_\perp^2}{p_\perp^2}\int_z^1 \frac{dy}{y^2} C_F\left(\frac{1+y^2}{1-y}\right)_+ D_{NS}^{h_1,h_2}(\frac{z_1}{y};\frac{z_2}{y}) \\ &+ \frac{\alpha_s}{2\pi}\int^{Q^2}\frac{dl_\perp^2}{l_\perp^2}\int_{z_1}^{1-z_2}\frac{dy}{y(1-y)} \\ &\times C_F\frac{1+y^2}{1-y} D_{NS}^{h_1}\left(\frac{z_1}{y}\right) D_g^{h_2}\left(\frac{z_2}{1-y}\right), \end{aligned} \quad (3)$$

where the leading order fragmentation functions are defined as matrix elements of field operators. The operator expressions for the single inclusive fragmentation functions may be obtained from Ref. [1]. The $+$ functions indicate the inclusion of both real and virtual contributions. The full DGLAP evolution equation is obtained by substituting the LO fragmentation functions in the above equation with the expressions for the NLO fragmentation functions and interating the process. For phenomenological applications of this formula see Refs. [1, 2]

[1] A. Majumder and X.-N. Wang, Phys. Rev. D. *to appear* (2004).
[2] A. Majumder, J. Phys. G. *to appear* (2004).